# Achieving Optimal Degrees of Freedom in Multi-Source Interfering Relay Networks With Multiple Antennas

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Abstract—We study multi-stream transmission in the  $K \times$  $N \times K$  channel with *interfering relays*, consisting of K source– destination (S-D) pairs and N half-duplex relays in-between them, where the source and destination nodes are equipped with M antennas. We introduce a new achievability scheme, termed multi-stream opportunistic network decoupling (MS-OND), to enable us to transmit  $1 \le S \le M$  data streams per S–D pair, operating in virtual full-duplex mode. More specifically, our protocol intelligently integrates multi-source random beamforming (MS-RBF) for the first hop and opportunistic interference alignment (OIA) for the second hop into our multi-source interfering relay by taking into account the total interference level (TIL). A subset of relays using alternate relaying is opportunistically selected in terms of producing the minimum TIL, thereby resulting in network decoupling. As our main result, it is shown that under a certain relay scaling condition, the MS-OND protocol achieves SK degrees of freedom even in the presence of interfering links among relays. Numerical evaluation is also shown to validate performance of the proposed MS-OND. Our protocol basically operates in a fully distributed fashion along with local channel state information.

#### I. INTRODUCTION

Research on the K-user two-hop relay-aided interference channel, consisting of K source–destination (S–D) pairs and N helping relays located between the S–D pairs, so called the  $K \times N \times K$  channel, has been attracting academic attention. In [1], under assumptions of no inter-relay interference and fullduplex relays, an achievable scheme, named aligned network diagonalization, was proposed for the network model, showing that the optimal degrees of freedom (DoF) can be obtained in a high signal-to-noise ratio (SNR) regime. Under more practical assumptions that there is an interfering signal between relays and the relays are half-duplex, a recent study introduced *opportunistic network decoupling (OND)* [2] that asymptotically achieves K DoF under a certain relay scaling condition in the  $K \times N \times K$  channel by exploiting the multiuser diversity gain on the usefulness of fading.

As a natural extension of the single-antenna configuration in [1], [2], we study the  $K \times N \times K$  channel with interfering half-duplex relays, where the sources and destinations are equipped with  $M \ge 1$  antennas and each source transmits  $1 \le S \le M$  data streams. Extension to this multi-stream scenario is not straightforward since more challenging and sophisticated interference management strategies are accompanied in the network model. We propose a *multi-stream* OND (MS-OND) protocol such that the set of relays receives and forwards multiple data streams per S-D pair with virtual fullduplex operation. Our protocol intelligently integrates *multi-source random beamforming (MS-RBF)* for the first hop and *opportunistic interference alignment (OIA)* [3] for the second hop into the network decoupling framework. As our main result, it is shown that in a high SNR regime, the MS-OND protocol achieves SK DoF when the number of relays scales faster than SNR<sup>3SK-S-1</sup>. The detailed description and all the proofs are omitted due to page limit.

### II. SYSTEM AND CHANNEL MODELS

K sources and the corresponding destinations are equipped with  $M \ge 1$  antennas and the relays are equipped with a single antenna. There is no direct communication path between each S–D pair, and thus each source transmits  $1 \le S \le M$ data streams to its destination through multiple relays selected out of N relays. Each relay is assumed to fully decode, reencode, and forward one data stream. Unlike the work in [1], the N relays are assumed to interfere with each other in our model. All the channels are assumed to be Rayleigh, having zero-mean and unit variance. We assume the blockfading model, i.e., the channels keep constant during one block and independently vary for every block.

#### **III. ACHIEVABILITY RESULTS**

We introduce an MS-OND protocol, where 2S relays among N candidates are opportunistically selected for an S–D pair to forward S data streams in the sense of having a sufficiently small amount of total interference level (TIL). In our protocol, each source employs MS-RBF to confine the interference from other beams for the first hop, while each destination employs OIA to confine the interference leakage from other pairs' transmission for the second hop. The proposed protocol is basically performed by utilizing the channel reciprocity of time-division duplexing (TDD) systems.

**Overall procedure:** (a) Initialization phase: All the sources and destinations generate random beamforming vectors and interference space along with MS-RBF and OIA, respectively. Two relay sets  $\Pi_1$  and  $\Pi_2$  are selected to alternately receive and forward data streams; (b) Odd time slots: K sources transmit data to the relays in  $\Pi_1$ . Meanwhile, the relays in  $\Pi_2$  forward the encoded data received from the sources in the previous even time slot to the destinations; (c) Even time slots: K sources transmit data to the relays in  $\Pi_2$ . The relays in  $\Pi_1$  forward the encoded data received from the sources in the previous odd time slot to the destinations; (d) Reception at the destinations: All the destinations receive and decode the symbols forwarded from either  $\Pi_1$  or  $\Pi_2$  by using zeroforcing detection.

**Opportunistic relay selection:** 1) The first relay set selec*tion:* Relay  $\mathcal{R}_n$  for  $n \in \{1, \dots, N\}$  computes SK scheduling metrics  $L_{n,(k,s)}^{\Pi_1}$  for stream  $s \in \{1, \cdots, S\}$  of S–D pair  $k \in \{1, \cdots, K\}$ , consisting of the interference from other beams caused by MS-RBF for the first hop and the interference leakage from other pairs' transmission caused by OIA for the second hop. By using a distributed timer method, SK relays with small scheduling metrics are opportunistically selected as  $\Pi_1$ ; 2) The second relay set selection: Relay  $\mathcal{R}_n$  for  $n \in \{1, \dots, N\} \setminus \Pi_1$  can compute the inter-relay interference from  $\Pi_1$ , denoted by  $L_n^{(r)}$ , since  $\Pi_1$  has already been selected in the  $\Pi_1$  selection step. Thus, the relay  $\mathcal{R}_n$  computes SKTILs  $L_{n,(k,s)}^{\Pi_2} = L_{n,(k,s)}^{\Pi_1} + L_n^{(r)}$  as another scheduling metric, thus capturing the inter-relay interference. Then, SK relays are opportunistically selected as  $\Pi_2$ . By virtue of the channel reciprocity of TDD systems, the interference from  $\Pi_2$  to  $\Pi_1$ can also be confined, thereby enabling the virtual full-duplex operation.

The achievability results of our protocol are shown in the following two theorems.

**Theorem 1**: Suppose that the MS-OND protocol with alternate relaying is used for the  $K \times N \times K$  channel with interfering relays. Then, the total DoF is bounded by SK if N scales faster than SNR<sup>3SK-S-1</sup>.

**Theorem 2**: The decaying rate of the average TIL is lowerbounded by  $N^{\frac{1}{3SK-S-1}}$ .

## IV. NUMERICAL RESULTS

In this section, we perform computer simulations to validate the achievability results of the proposed MS-OND protocol in the  $K \times N \times K$  channel with interfering relays. In our simulations, the channel coefficients are generated  $1 \times 10^5$ times for each system parameter. It is assumed that M = 4for all the simulations.

In Fig. 1, the log-log plot of the average TIL versus N is shown for the  $K \times N \times K$  channel according to various system parameters, where the MS-OND with alternate relaying is used and  $(K, S) = \{(2, 1), (3, 1), (2, 2)\}$ . It is observed that the TIL tends to linearly decrease with N in a large N regime. In this figure, the dotted lines are also plotted from the theoretical results in Theorem 2 with a proper bias to check the slope of the TIL. We can see that the decaying rates of TIL are consistent with the relay scaling law condition in Theorem 1. More specifically, as N increases, the TIL is reduced with the slope of  $\frac{1}{4}$ ,  $\frac{1}{7}$ , and  $\frac{1}{9}$  for  $(K, S) = \{(2, 1), (3, 1), (2, 2)\}$ , respectively.

Figure 2 illustrates the achievable sum-rates for the MS-OND protocol with alternate relaying versus snr (in dB scale) when K = 2, N = 500, and S = 1, 2, 3 in the  $K \times N \times K$  channel with interfering relays. For comparison, the performance for the MS-OND without alternate relaying as a baseline is also plotted in the figure. We can see that the MS-OND without alternate relaying case outperforms the alternate



Fig. 1: The average TIL versus N for various system parameters in the  $K \times N \times K$  channel with interfering relays.



Fig. 2: The achievable sum-rates versus SNT when K = 2, S = 1, 2, 3, and N = 500 in the  $K \times N \times K$  channel with interfering relays.

relaying case in some medium or high SNR regimes. This is because for finite N, the sum-rates for the alternate relaying case tend to be saturated faster than no alternate relaying case with increasing SNr due to more residual interference in each dimension. It is also seen that higher S yields better sum-rates in the low SNr regime but gets saturated earlier with increasing SNr.

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